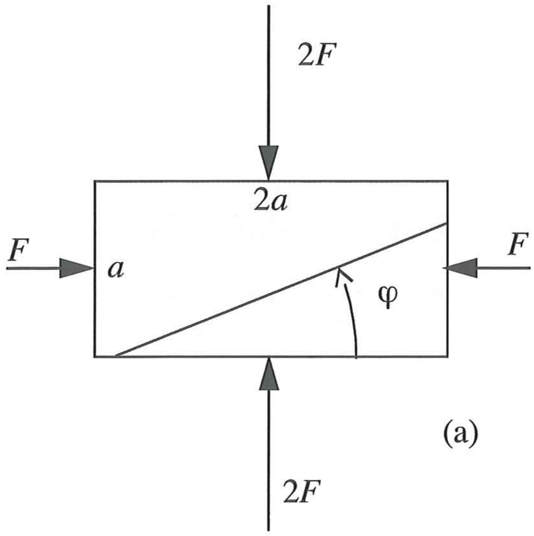
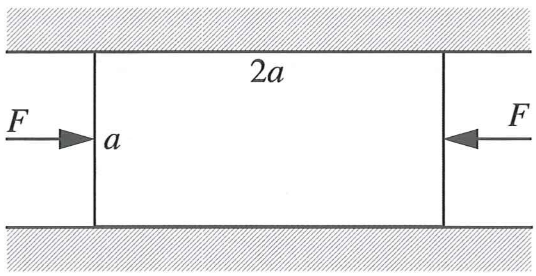
Tentamen
SOLID MECHANICS (NASM)
April 16, 2012, 14:00–17:00 h

Question 1 Assume we have a block of material having dimensions $a \times 2a$ and being subjected to plane strain deformation perpendicular to the plane of the paper.



(a)

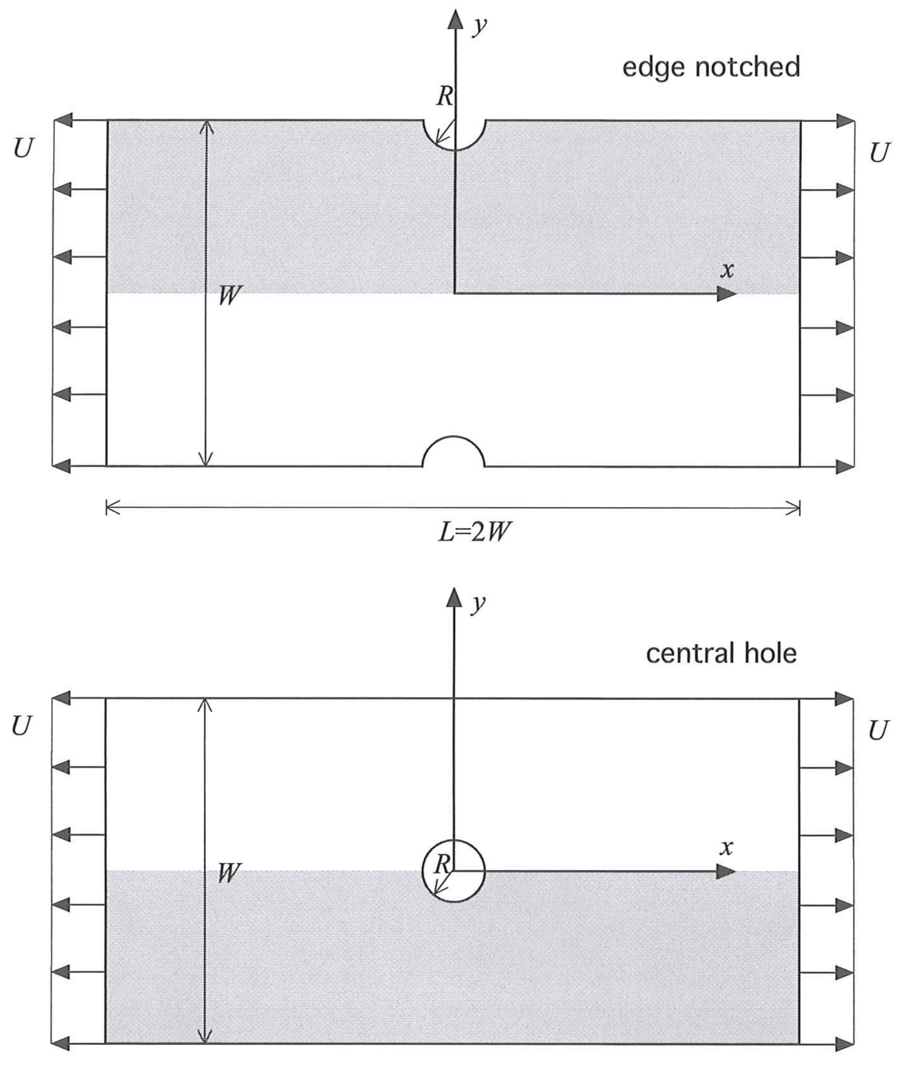


(b)

a. Consider the situation of figure (a) where the block is subjected to a horizontal compressive force F and a vertical force $2F$. Assume that the loading is applied such that the stress state inside the material is uniform. What is the resolved shear stress on a plane inclined at the angle φ as shown? Is this the maximum shear stress in the material?

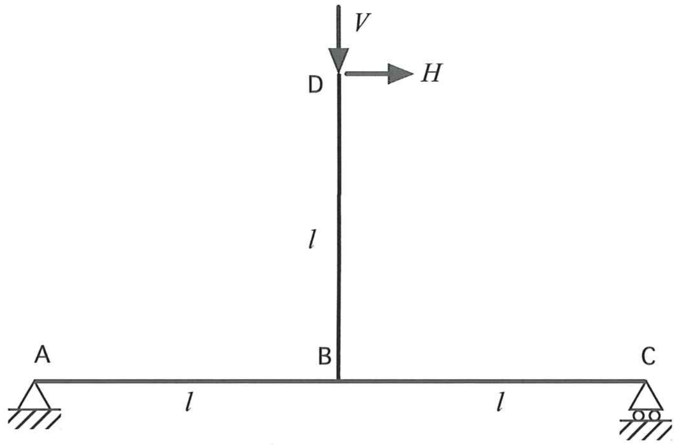
b. In the loading situation of figure (b), the block is subjected to the same horizontal force F but is constrained in its vertical deformation by means of rigid walls. If the material response is elastic, determine all components of the stress tensor.

Question 2 Two rectangular plates of W by $2W$ are perforated with a hole of radius R in different ways: one with two half-circular notches at the edges, the other with a central hole, as shown in the figure below.



The total amount of material left is the same, and in fact the gray areas indicating symmetric halves are identical. Nevertheless, the two plates will respond to an applied displacement U in different ways (even if they have the same uniform material properties). Apparently something in the entire set of governing equations is different; the question is: what?
Support your explanation with mathematical statements.

Question 3 A $\perp$-like spring consists of three legs having length l and bending stiffness EI. It is mounted on simple supports at the ends A and C, and is loaded at the remaining end D. A horizontal force H gives rise to a horizontal displacement u_D, a vertical force by a vertical displacement v_D, thus giving rise to two stiffnesses. The question is: which of the two, H/u_D or V/v_D, is largest.



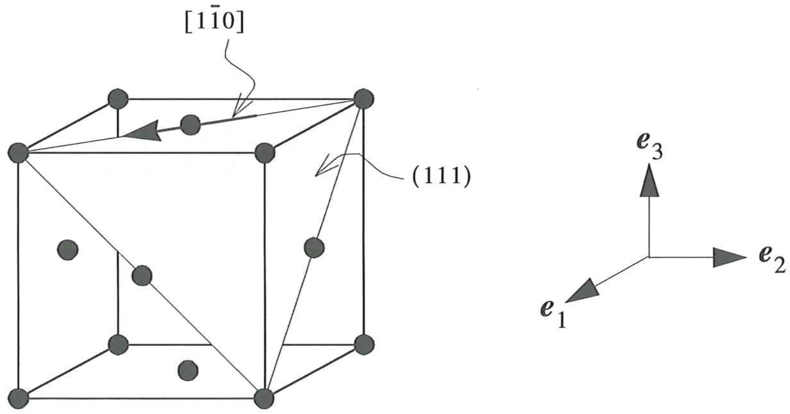
a. Compute the vertical stiffness V/v_D, neglecting (as usual) the axial deformation of the bar BD.

b. Determine the distribution of the bending moment in ABC caused by the horizontal force H. Specify the boundary conditions along ABC (these are needed to solve the beam equation and are helpful in using the 'forget-me-nots').

c. Compute the horizontal stiffness H/u_D.

PS. You may either solve the beam equation or use the 'forget-me-nots'. In either case, make use of (point) symmetry to make your life easier.

Question 4 An fcc crystal is known to have 12 slip systems: 3 different slip directions on each of 4 equivalent slip planes. One example is indicated in the figure below by means of Miller indices.



In terms of components with respect to the orthonormal basis $\{\boldsymbol{e}_i\}$ $(i = 1,2,3)$ this $(111)[1\bar{1}0]$ slip system is characterized by the slip plane normal vector $\boldsymbol{m}$ and slip direction $\boldsymbol{s}$ given as[1]

$$m = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad s = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

[1]Note that an overbar in Miller indices corresponds to a minus sign in vector notation.

The following table lists all slip systems in terms of Miller indices and their usual names in the materials science community:

name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	$(1\bar{1}1)$	$(1\bar{1}1)$	$(1\bar{1}1)$
direction	$[\bar{1}01]$	$[0\bar{1}1]$	$[\bar{1}10]$	$[\bar{1}01]$	[011]	[110]
name	A2	A6	A3	C5	C3	C1
plane	$(\bar{1}11)$	$(\bar{1}11)$	$(\bar{1}11)$	$(11\bar{1})$	$(11\bar{1})$	$(11\bar{1})$
direction	$[0\bar{1}1]$	[110]	[101]	$[\bar{1}10]$	[101]	[011]

a. Let this crystal be subjected to uniaxial tension in the [001] direction, i.e., $\boldsymbol{\sigma} = \sigma \boldsymbol{e}_3 \otimes \boldsymbol{e}_3$. For each slip system, compute the ratio between the resolved shear stress and the applied stress σ (also known as the Schmid factor).

Hint: under this loading, 8 out of 12 slip systems will be activated.

b. Show that tension in the [011] direction is represented by the stress components

$$[\sigma_{ij}] = \frac{1}{2}\sigma \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

with respect to $\{\boldsymbol{e}_i\}$.

c. How many slip systems are active when the crystal is subjected to uniaxial tension in the [011] direction?

Question	# points
1	2+2=4
2	3
3	2+2+2=6
4	3+1+2=5

Exam grade = (# points + 2)/2.1

Best Case
3
2
6
4 +
15

Worst Case:
1 → 3?
1 → 0?
4 → 5?
2 → 3?
8 11 → 6, 2. ..